NAME OF MATHLYMPIAN: ______________________________

ANNUAL MATHLYMPICS

FOR PRIMARY SCHOOLS

Vault Round (Exemplar)

1 hour 30 minutes

Instructions to Mathlympians

1. Do not open the booklet until you are told to do so.
2. Attempt ALL 28 questions.
3. Diagrams are not drawn to scale.
4. Write your answers neatly on the ANSWER SHEET provided.
5. Marks are awarded for correct answers only.
6. **No calculators may be used.**

Questions in Section A carry 2 marks each, questions in Section B carry 4 marks each and questions in Section C carry 5 marks each.

This paper consists of Questions 1 to 28 on pages 1 to 10.

Section A

Each of the questions 1 to 10 carries 2 marks.

1. Abel's age in 2019 was equal to the sum of the digits of his year of birth. If Abel is an adult, how old was Abel in 2018?

2. Given that $6^2 = 6 \times 6$; $6^3 = 6 \times 6 \times 6$; $6^4 = 6 \times 6 \times 6 \times 6$; etc, what is the sum of the last two digits of 6^{2018}?

3. Find the sum of all the numbers less than 1000 that are divisible by both 5 and 7.

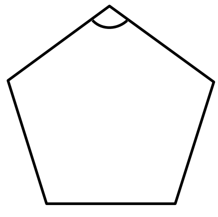
4. Three friends, Caria, Dennis and Emma went out shopping for packets of candies. Caria took half of all the packets, Dennis took a third of the remaining packets and Emma took the rest. As Caria did not bring any money, Dennis and Emma split the bill equally. The next day, Caria came with $15 to return to her friends. How much should she return to Emma?

5. Fifty students with index numbers 1 to 50 are waiting in a room. A teacher comes in and calls out the first 3 prime numbers in sequence. Students with index numbers corresponding to the called-out prime numbers and their multiples, left the room. How many students remained in the room?

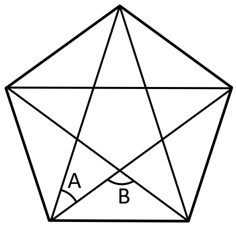
6. A number is a palindrome if it is the same when read backwards as forwards. An example of a palindrome is 12321. Find the largest palindrome that can be formed from the product of two 2-digit numbers.

7. The first five terms of a sequence is 52 45 38 31 24 .
Write an expression to find the nth term of this sequence in the format $\square - \square n$.

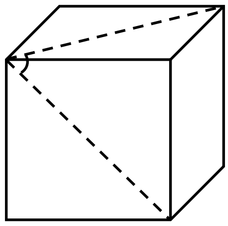
8. The figure shown is a polygon known as a pentagon.
Because each of the sides are equal, we call it a regular pentagon.
Find one of the interior angles of a regular pentagon.



9. The figure shows a star drawn inside a regular pentagon.
   Find the difference of ∠A and ∠B.



10. What is the angle inside the cube that is between the 2 diagonals as shown below?



Section B

Each of the questions 11 to 20 carries 4 marks.

11. Solve $\dfrac{4 \times A + \frac{1}{2} + 3 \times 2018 + 2A \times 22}{4 \times A + \frac{1}{6} + 2 \times 1009 + A \times 12} \times 3$

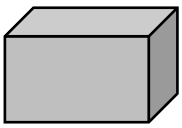
12. The product of a 6-digit number, $\overline{1ABCDE}$ and 3, is $\overline{ABCDE1}$, that is, $\begin{array}{r} 1\text{ABCDE} \\ \times \quad\quad 3 \\ \hline \text{ABCDE1} \end{array}$.
    What is the 5-digit number $\overline{ABCDE}$?

13. How many 3-digit numbers which are multiples of 7 end with the digit 4?

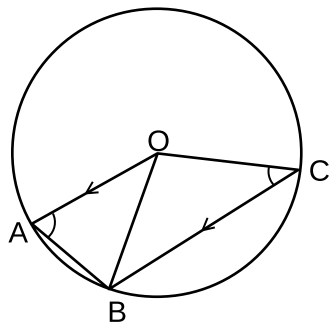
14. When two consecutive prime numbers are separated by a single number, they are known as prime pairs. An example of a prime pair is 29 and 31. What is the highest common factor of all the numbers found between each 3-digit prime pairs?

15. Jan's father is thrice the age of Jan. In n years' time, Jan's father will be twice Jan's age then. How many times is Jan's father's age in n years' time compared to Jan's age now?

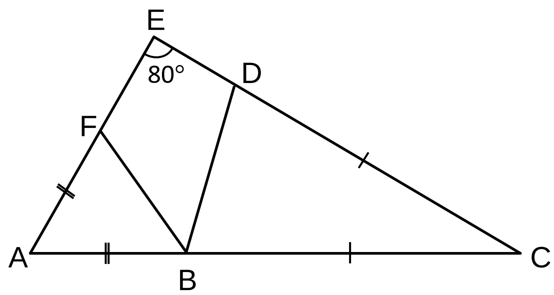
16. The volume of a solid cuboid is 144 cm^3. The perimeter of the top face of the cuboid is 22 cm. If each side of the cuboid is a whole number of centimetres and longer than 2 cm, find the height of the cuboid.



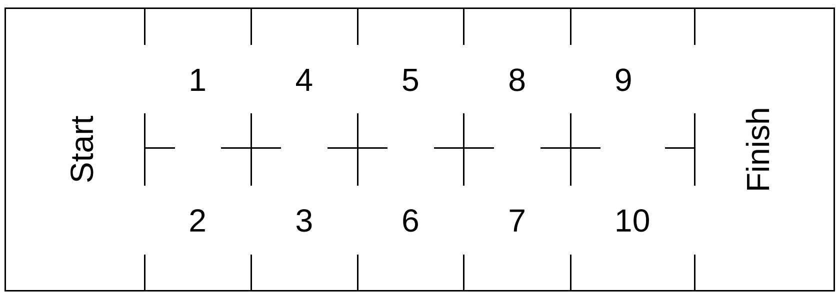
17. In the diagram, O is the centre of the circle and A, B and C are points on the circumference.
OA is parallel to CB. The ratio of ∠OAB and ∠OCB is 5 : 2.
Find ∠AOC.



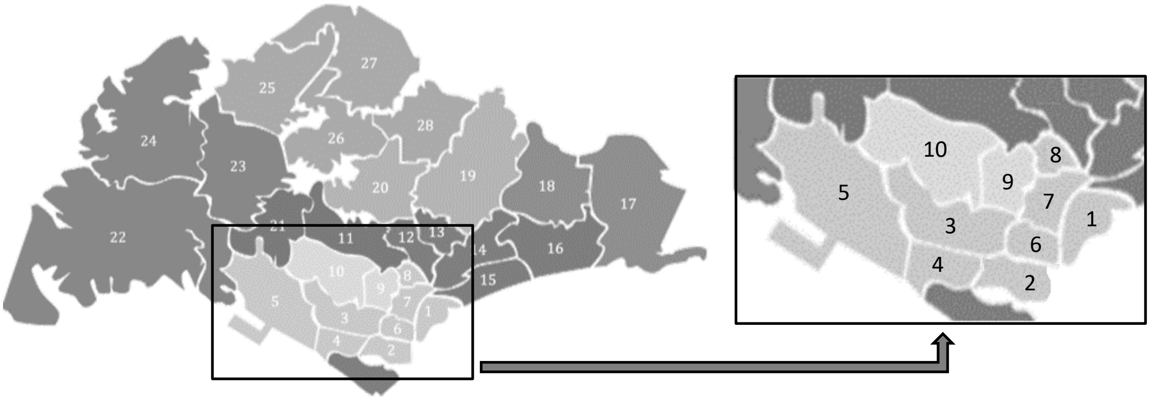
18. Given that AB = AF and BC = CD and ∠DEF = 80°.
Find ∠DBF.



19. The diagram below shows the layout of 10 connected rooms. How many ways are there to get from the Start corridor to the Finish corridor if you can only move from a room of a smaller number to one with a larger number?



20. Below is Singapore's 28 District Code map.



At most how many ways are there of colouring the central district (No. 1 – 10) using three different colours such that adjacent districts do not share the same colour?

Section C

Each of the questions 21 to 28 carries 5 marks.

21. On the first day, Elisa read $\frac{1}{6}$ of a story book. The next day, she finished another 40 pages. On the third day, what she read was 1.4 times as many pages as the first 2 days combined. She finished the last $\frac{1}{3}$ of the book on the fourth day. How many pages are there in the book?

22. Amy planned to cover the 8 m by 8 m floor of her living room with a square carpet in the middle and with wooden tiles along the borders. The carpet and wooden tiles cost $40/m^2 and $15/m^2 respectively and she paid $1360 in total. What is the width of the wooden tiled border?

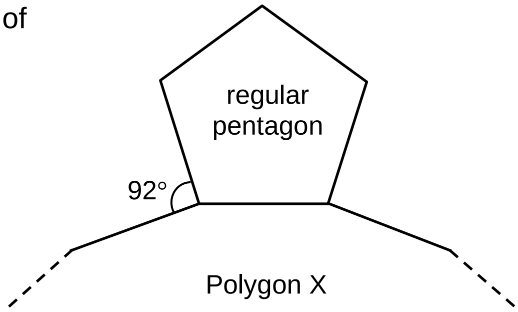
23. If we write out the digits of all the positive integers into a sequence of digits starting from 1, that is, 1 2 3 4 5 6 7 8 9 1 0 1 1 1 2 …, we will encounter the first of three consecutive 5s in the 100th position of the sequence. At which position will we encounter the first of five consecutive 5s?

24. Find how many consecutive zeros there are at the end of the product of the following multiplication: $166 \times 172 \times 178 \times \ldots \times 598$.

25. In $\frac{1}{A+\frac{1}{B+\frac{1}{C}}} = \frac{7}{10}$, each letter represents a different digit. What is the values of A + B + C?

26. Polygon X is a regular polygon. It touches one side of a regular pentagon as shown in the figure.

Find the number of sides Polygon X has.



27. A 200-m long train travelling at the speed of 30 km/h closed up on a jogger at 7:45 a.m. It took 30 seconds for the train to completely pass the jogger. At 7:54 a.m., the train came upon a cyclist. It took the train 15 seconds to completely pass the cyclist. At what time will the cyclist and the jogger meet?

28. Nonograms are logic puzzles where the numbers along the top indicate the sequences and numbers of consecutive cells down the respective columns which have to be shaded; and the numbers along the left indicate the sequences and numbers of consecutive cells along the respective rows which have to be shaded. A sample nonogram and its solution is shown below.

	2 2	4	1	2	2
1 1					
2 2					
3					
2					
2					

→

	2 2	4	1	2	2
1 1	■				■
2 2	■	■		■	■
3		■	■	■	
2	■	■			
2	■	■			

Shade the nonogram below.

	1 1 2 3	3 4	1 1 1 2	2 1 2	1 7	4 2	4 1	3	4 1 1	2 2 2
1 1 1 1										
1 1 1 1										
4 1 1										
1 2 1										
1 1 3 1										
2 2 4										
1 1 2 1										
2 2 3										
6 1										
1 1 3 2										

End of Paper

NAME OF MATHLYMPIAN: ______________________________

ANNUAL MATHLYMPICS

FOR PRIMARY SCHOOLS

Sprint Round (Examplar)

30 minutes

Instructions to Mathlympians

1. Do not open the booklet until you are told to do so.
2. Attempt as many questions as you can within the given time.
3. Questions are NOT arranged in increasing order of difficulty.
4. Diagrams are not drawn to scale.
5. Write ALL your answers neatly on the answer sheet provided.
6. Marks are awarded for correct answers only.
7. **You may use the Casio calculator provided.**

All questions carry 1 mark each.

Sprint Section
Each of the questions 1 to 30 carries 1 mark.

1. My age is 45 divided by $\frac{1}{5}$ of my age. How old am I?

2. If you multiply all the odd numbers between 1 and 2018, what is the last digit of the product?

3. A student scored an average of 23 marks for several weekly tests. After the final test, his average became 22.25. What did he score on his final test if his score on every test was between 18 and 30 marks?

4. From all the 4-digit numbers which can be formed using the digits 3, 4, 6, and 7 without repetition, how many are exactly divisible by 44?

5. What is the smallest 3-digit square number which is also the difference of two other square numbers?

6. What is the largest possible number of prime numbers that can occur in a set of 20 consecutive numbers greater than 50?

7. An animal trainer has 5 tigers and 4 lions. He would like to line them up so that no two lions and no two tigers are together. In how many ways can he line up his animals?

8. The ratio of the lengths of the two diagonals of rhombus ABCD is 3 : 4.
   The sum of their lengths is 56 cm. What is the perimeter of rhombus ABCD?

9. A cubic container can hold 1728 litres of water. How many cubes of sides 25 cm can fit inside the cubic container?

10. In the equation $\frac{(x+1)}{1} + \frac{(x+2)}{2} + \cdots + \frac{(x+100)}{100} = 100$, what is x?

11. A couple has 7 children. Each child is born exactly 18 months apart. If the sum of the ages of the three youngest children is 29 years, what is the sum of the ages of the three oldest children?

12. A total of 60 blackbirds were sitting in three trees. Suddenly, 6 blackbirds flew away from the first tree, 8 from the second, and 4 from the third. After that, there were twice as many birds in the second tree as the first, and twice as many in the third tree as the second. How many birds were originally in the second tree?

13. A group of men and women went shopping. Each man in the group spent $200, each woman spent $160, and the total they spent was $2640. If there were an odd number of people in the group, how many of them were men?

14. Norman wrote down nine numbers in increasing order. The middle number is the average of all nine numbers. The average of the first five numbers is 27 and the average of the last five numbers is 49. What is the sum of all the numbers?

15. The sum of the digits in a 4-digit number is 9. None of the digits is 0. The 4-digit number is a multiple of 5 and is greater than 1905. What is the digit in the Tens' place?

16. A number is called stuttering if all its digits are 1. How many whole numbers are there, between 1 and 100 000, which can be multiplied by 33 to give a stuttering product?

17. A large box contains thousands of marbles of 20 different colours all mixed together. The marbles are sold only by weight as they are being scooped out randomly. Each marble weighs 5g. What is the weight of all the marbles you must buy to ensure that you get at least 100 marbles of the same colour? (Give your answer in g.)

18. If 97 336 small cubes were put together to form a huge cube, how many of these small cubes cannot be seen from the outside?

19. If $\frac{2x-y}{x+y} = \frac{2}{3}$, what is $\frac{x}{y}$?

20. The numbers represented by a, b, c, d, and e are greater than 0 and satisfy the conditions ab = 2, bc = 3, cd = 4, and de = 5. What is $\frac{e}{a}$?

21. Two litres of fruit juice containing 10% sugar are mixed with 3 litres of vegetable juice containing 15% sugar. What percentage of the mixture now consists of sugar?

22. For every pot of roses that Jimmy delivered to a nursery, he was paid $1.20. But if he damaged any pot, he would not be paid for that pot's delivery and instead he had to pay $2.00 as penalty. If he was paid a total of $568 to deliver 500 pots of roses, how many pots of roses did he damage?

23. Cathy and Roy had some candies. If Cathy and Roy ate 8 candies and 4 candies respectively each day, Cathy would still have 50 candies when Roy had eaten up all his candies. If Cathy and Roy ate 4 candies and 8 candies respectively each day, Cathy would still have 200 candies when Roy had eaten up all his candies. How many candies did each of them have?

24. A 2-digit number is called reflexive if it is 27 more than the number obtained by reversing its digits. How many 2-digit numbers, which do not have 0 as either of their digits, are reflexive?

25. How many 3-digit numbers are odd, exactly divisible by 3, and less than 456?

26. A 2-digit number is called fascinating if the number obtained by reversing its digits is 75% larger than the original number. For example, 24 is fascinating because 42 is 75% larger than 24. How many two-digit numbers are fascinating?

27. A chess board has 32 black squares and 32 white ones. In how many ways can you choose one white and one black square on a chess board so that they are neither in the same row nor in the same column?

28. In the rectangle ABCD, M and N are the midpoints of AD and BC, respectively, and AC intersects MB and DN at P and Q, respectively.
If AD is 24 cm and AB is 18 cm, what is the area of MPQD in cm^2?

29. A container has a square base with a side of 8 cm. Twelve metal 4-cm cubes are placed inside in the container. Water is then poured into the container until it is $\frac{5}{6}$ full. When all the metal cubes are removed, the water level dropped to $\frac{2}{3}$ the height of the container. Find the height of the container.

30. Eleanor was travelling from City F to City G at a uniform speed. She drove past Rachel who was travelling at a uniform speed of 84 km/h in the opposite direction. One and a half hours later, Eleanor reached City G while Rachel was still 39 km away from City F. If Eleanor took 4 hours to travel from City F to City G, what was the distance between the two cities?

Exemplar Vault Section_Solutions

1. If Abel was born in 2018, he would be 1 year old in 2019 and the digit sum = 2 + 1 + 8 = 11
   If Abel was born in 2017, he would be 2 years old in 2019 and the digit sum = 2 + 1 + 7 = 10
   If Abel was born in 2016, he would be 3 years old in 2019 and the digit sum = 2 + 1 + 6 = 9
   :
   If Abel was born in 2013, he would be 6 years old in 2019 and the digit sum = 2 + 1 + 3 = 6
   But Abel cannot be 6 years old because he is an adult.

   If Abel was born in 1999, he would be 20 years old in 2019 and the digit sum = 28
   If Abel was born in 1998, he would be 21 years old in 2019 and the digit sum = 27
   :
   If Abel was born in 1995, he would be 24 years old in 2019 and the digit sum = 24
   Abel's age in 2018 was 24 – 1 = 23 years old.

2. From 6^2 onwards, the last two digits occur according to the sequence 36, 16, 96, 76 and 56, and then the sequence repeats. Since the sequence is a set of 5 numbers and bearing in mind to account for the first 6^1 = 6, then (2018 – 1) ÷ 5 has a remainder of 2. Therefore, the last two digits of 6^{2018} is 16. Sum of 1 and 6 = 7.

3. A number that is divisible by both 5 and 7 is divisible by 35. There are 28 multiples of 35 that is less than 1000. The sum of these multiples = (1+2+3+…+28) × 35 = $\frac{29\times28}{2}$ × 35
   = 14210.

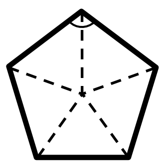
4. Caria took $\frac{1}{2}$ of the packets.
   Dennis took $\frac{1}{3}\times\frac{1}{2}=\frac{1}{6}$ of the packets and Emma took $\frac{1}{2}-\frac{1}{6}=\frac{1}{3}$ of the packets.
   If \$15 is for half of the cost, then each of Dennis and Emma must have each paid \$15 at the beginning although Emma should have only paid a third of \$30, that is \$10. Therefore, Caria should return Emma \$5.

5. 14 students with index numbers 1, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47 and 49 remained.

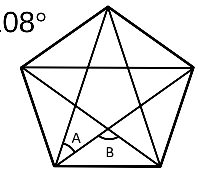
6. The largest product of two 2-digit numbers is 9801, (that is 99 × 99). Since the required product is a palindrome, then it has to be 9XX9 where X represents an unknown digit. Since the number ends with 9, then the two factors can only be (91 and 99) , (93 and 93) or (97 and 97). The only pair which gives a palindrome product is 91 and 99. Hence, the product is 91 × 99 = 9009.

7. The formula to find any term is 52 – 7(n – 1), which is simplified to 52 – 7n + 7 = 59 – 7n.

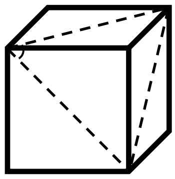
8. The pentagon can be divided into 5 triangles.
   The sum of all the angles of the 5 triangles = 180 × 5 = 900°
   The sum of all the interior angles of the pentagon = 900 – 360 = 540°
   Each interior angle = 540 ÷ 5 = 108°.



9. $\angle B$ = vertically opposite to interior angle of a pentagon = 108°
180° – 108° = 72°
$\angle A$ = 180° – 72° – 72° = 36°
108° – 36° = 72°
The difference between $\angle A$ and $\angle B$ is 72°.



10. The triangle inside the cube is an equilateral triangle since the diagonals of all the faces of the cube are equal. Hence, the angle is 60°



11. $$\frac{4 \times A + \frac{1}{2} + 3 \times 2018 + 2A \times 22}{4 \times A + \frac{1}{6} + 2 \times 1009 + A \times 12} \times 3 = \frac{48A + 6 \times 1009 + \frac{1}{2}}{16A + 2 \times 1009 + \frac{1}{6}} \times 3$$

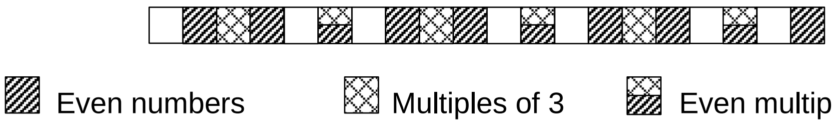
$$= \frac{3\left(16A + 2 \times 1009 + \frac{1}{6}\right)}{\left(16A + 2 \times 1009 + \frac{1}{6}\right)} \times 3$$

= 3 × 3 = 9

12. Since the product of the digit E and 3 end with the digit 1, E must be 7. So, 7 × 3 = 21.
Then, since D × 3 + 2 = □7, or D × 3 = □5, so D must be 5. That is, 5 × 3 = 15.
Then, since C × 3 + 1 = □5, or C × 3 = □4, so C must be 8. That is, 8 × 3 = 24.
Then, since B × 3 + 2 = □8, or B × 3 = □6, so B must be 2. That is, 2 × 3 = 6.
Then, since A × 3 = □2, so A must be 4. That is, 4 × 3 = 12.
Hence, the 5-digit number ABCDE is 42857.

13. The largest 3-digit multiple of 7 is the 142nd multiple, which is 994. The smallest 3-digit multiple is the 15th multiple, which is 105. The multiples of 7 among them that end with the digit 4 are the 22nd, 32nd, 42nd, …, 142nd multiples of 7. There are 13 of them.

14. Let the row of squares below show where even numbers and multiples of 3 are found in a series of consecutive numbers.



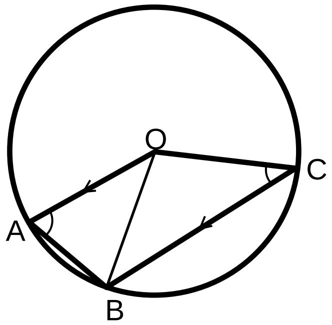
The white boxes represent where numbers which are neither even nor multiples of 3 can be positioned. Some of these white boxes represent prime numbers.
Where two white boxes are separated by a single box, that single box always represents even multiples of 3. The highest common factor of these even factors of 3 is 6.

15. Let Jan's age now be represented by j. Jan's father's age now is therefore $3j$.
$3j + n = 2(j + n)$ or $3j + n = 2j + 2n$. Therefore, $j = n$.
Jan's father's age in n years' time is $3j + n$ or $4j$ whereas Jan's age now is j.
Therefore, Jan's father's age in n years' time is 4 times the age of Jan now.

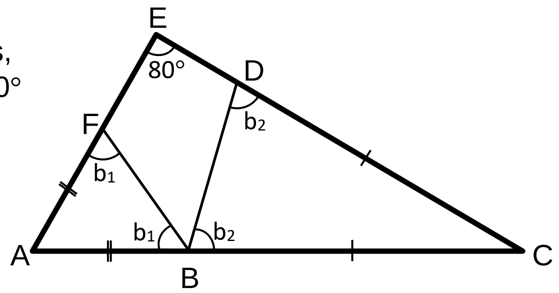
16. $144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$
Sum of length and breadth of cuboid = $22 \div 2 = 11$ cm
$11 = (3 \times 3) + 2$ or $11 = (2 \times 2 \times 2) + 3$
Since the sides are greater than 2 cm, the length must be $(2 \times 2 \times 2)$ and the breadth must be 3 cm. Therefore, Volume, $144 = \underbrace{2 \times 2 \times 2}_{\text{length}} \times \underbrace{3}_{\text{breadth}} \times \underbrace{2 \times 3}_{\text{height}}$

The height of the cuboid is <u>6 cm</u>.

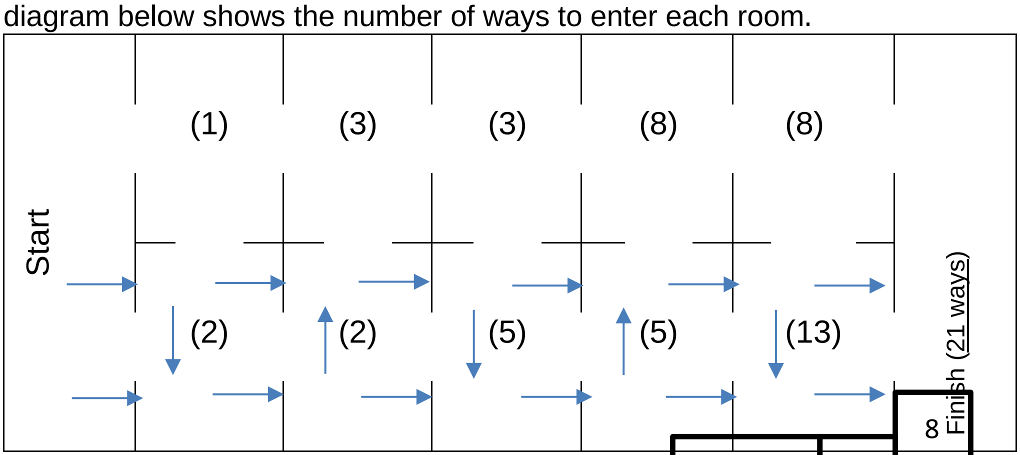
17. Let $\angle OAB$ = 5 units and $\angle OCB$ = 2 units.
Since triangles OAB and OBC are isosceles triangles,
then $\angle OBA$ = 5 units and $\angle OBC$ = 2 units,
and $\angle AOB$ = 2 units (alternate angles)
Consider ΔOAB, 5 units + 5 units + 2 units = 12 units = 180°,
therefore 1 unit = 15°.
Consider ΔOBC, 4 units + $\angle BOC$ = 180°.
Therefore, $\angle BOC = 180° - (4 \times 15) = 120°$
$\angle AOC$ = 2 units + 120° = 30° + 120° = <u>150°</u>



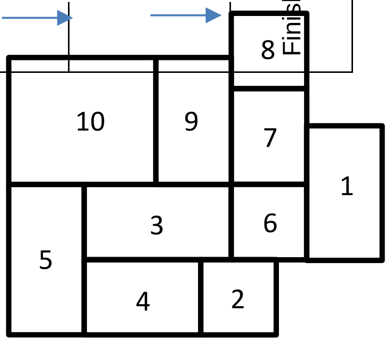
18. $\angle A + \angle C = 100°$
Since ΔAFB and ΔCBD are isosceles triangles,
then, $\angle A + \angle b_1 + \angle b_1 + \angle C + \angle b_2 + \angle b_2 = 360°$
$2 \times \angle b_1 + 2 \times \angle b_2 = 360° - 100° = 260°$
$\angle b_1 + \angle b_2 = 130°$
$\angle b_1 + \angle b_2 + \angle DBF = 180°$
$\angle DBF = 180° - 130° =$ <u>50°</u>



19. The diagram below shows the number of ways to enter each room.



20. Simplify the map into blocks.



Start from the most surrounded district, D3.

D3 → 3 ways
D9 → 2 ways
D10 → 1 way
D5 → 1 way
D4 → 1 way
D2 → 1 way
D6 → 1 way

D7 → 1 way
D8 → 1 way
D1 → 1 way
Total number of ways = 3 × 2 × 1 × 1 × 1 × 1 × 1 × 1 × 1 × 1 = 6 ways

21. Let B represent the number of pages of the book.

$\left(\frac{1}{6}B + 40\right) \times 2.4 = \frac{2}{3}B$

$\left(\frac{1}{6}B + 40\right) \times \frac{12}{5} = \frac{2}{3}B$

$\frac{2}{5}\text{B} + 96 = \frac{2}{3}B$

$\left(\frac{2}{3}B - \frac{2}{5}\ B\right) = 96$

$\frac{4}{15}B = 96$

$B = 360$

The book has 360 pages.

22. If the border is 1 m, then the square carpet at the center is 6 m by 6 m = 36 m^2
The cost = (36 × 40) + [(64 – 36) × 15] = 1440 + 420 = $1860

If the border is 2 m, then the square carpet center is 4 m by 4 m = 16 m^2
The cost = (16 × 40) + (64 – 16) × 15 = 640 + 720 = $1360

The wooden tiled border is 2 m in width.

23. For five consecutive 5s to line up, the digits must come from the number 555 and the first two digits of 556.
1^{st} to 9^{th} positions are taken up by 1-digit numbers. There are 180 positions after that which are taken up by 2-digit numbers from 10 to 99. The digits from 100 to 554 will take up [(554 – 99) × 3] = 1365 positions.
So far, 9 + 180 + 1365 = 1554 positions have been taken up.
The first digit of 55555 will take up the 1555^{th} position.

24. In the sequence, the numbers increase by 6. Hence, the numbers end with the digits 6, 2, 8, 4 and 0. There are [(598 – 166) ÷ 6 + 1] = 73 numbers. Of these numbers, those that are multiples of 5 are 190, 220, 250, 280, 310, 340, 370, 400, 430, 460, 490, 520, 550 and 580. There are 14 of them. Of these fourteen, 250, 400 and 550 are multiples of 25 which is (5×5), and 250 is also a multiple of 125 which is (5×5×5).
So, the number 5 is used 14 + 2 + 1 + 1 = 18 times to form these 14 numbers.
When each of these eighteen 5s are multiplied with an even number, it will become a multiple of 10, hence, the product will have 18 consecutive zeros at the end.

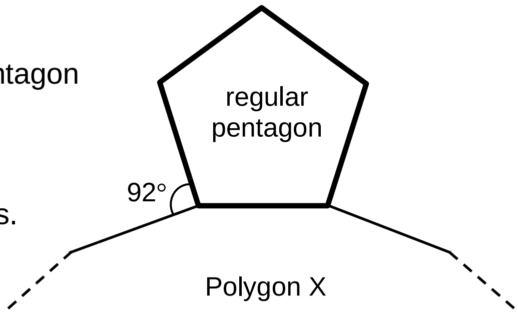
25. $$\frac{1}{A+\frac{1}{B+\frac{1}{C}}} = \frac{1}{A+\frac{1}{\frac{BC+1}{C}}} = \frac{1}{A+\frac{C}{BC+1}} = \frac{1}{\frac{ABC+A+C}{BC+1}} = \frac{BC+1}{ABC+A+C} = \frac{7}{10}$$

BC + 1 must be a multiple of 7 and BC is the product of two different digits.
BC could be 6, in which case B and C could be 1 and 6, or 2 and 3.
BC could not be 13 because B and C has to be single digit numbers.

BC could be 20, in which case B and C could be 4 and 5.
BC could be 27, in which case B and C could be 3 and 9.
BC could not be 34 or 41 because B and C has to be single digit numbers.
BC could be 48, in which case B and C could be 6 and 8.
BC could not be 55 or 62 or greater because B and C has to be single digit numbers.

If BC is 6, then ABC + A + C = 10 and A must be 1 → then ABC + A = 7 and C must be 3, B must be 2.
If BC is 20, then ABC + A + C = 30 and A must be 1 → then ABC + A = 21 and C must be 9 but C could only be 4 or 5.
If BC is 27, then ABC + A + C = 40 and A must be 1 → then ABC + A = 28 and C must be a a 2-digit number.
If BC is 48, then ABC + A + C = 70 and A must be 1 → then ABC + A = 49 and C must be a a 2-digit number.
Hence, we have the only combination of digits for A, B, C, which is 1, 2, 3 respectively.
A + B + C = 6

26. We already know that each interior angle of a pentagon is 108°, hence each interior angle of Polygon X = 360° – 108° – 92° = 160°.



An n-sided polygon can be divided into n triangles. (See Question 8)
Each interior angle = $\frac{180n-360}{n}$ = 160°
$160n = 180n - 360$
$20n = 360$
$n = 18$
Hence, Polygon X has 18 sides.

27. Train's speed = $\frac{30000}{60\times 60} = \frac{25}{3} m/s$
If the jogger had been stationary, the train would have taken 200 ÷ $\frac{25}{3}$ = 24s to completely pass him. Instead the train took 30s. This is because the train had to travel an addition 6 × $\frac{25}{3}$ = 50 m covered by the jogger. Jogger's speed = 50 ÷ 30 = $\frac{5}{3}$ m/s

Similarly, instead of taking 24s to completely pass the cyclist, the train took only 15s.
This is because the train travelled less 9 × $\frac{25}{3}$ = 75 m which was being covered by the cyclist coming from the opposite direction. Cyclist speed = 75 ÷ 15 = $5\ m/s$

From 7:45 to 7:54 is 9 min or 9 × 60 = 540s
Distance from the point the train met the jogger and the point when the train met the cyclist is 540 × $\frac{25}{3}$ = 4500 m.
In that time, the jogger jogged another 540 × $\frac{5}{3}$ = 900 m
Therefore, the distance between the jogger and cyclist is 4500 – 900 = 3600m

Since they are travelling towards each other, their combined speed is $\frac{5}{3} + 5 = \frac{20}{3}\ m/s$

$3600 \div \frac{20}{3} = 3600 \times \frac{3}{20} =$ 540s

540 ÷ 60 = 9 min

They will meet 9 minutes after 7:54 a.m., which is <u>8:03 a.m.</u>

28.

	1 1 2 3	3 4	1 1 1 2	2 1 2	1 7	4 2	4 1	3	4 1 1	2 2 2
1 1 1 1	■		■		■					■
1 1 1 1		■		■		■				■
4 1 1	■	■	■	■		■			■	
1 2 1		■			■	■			■	
1 1 3 1	■		■		■	■	■		■	
2 2 4	■	■		■	■		■	■	■	■
1 1 2 1		■			■		■	■		■
2 2 3	■	■		■	■		■	■	■	
6 1	■	■	■	■	■	■				■
1 1 3 2	■		■		■	■	■		■	■

Examplar Sprint Section_Solutions

1. Let my age be x years old.
   Then, $x = 45 \div \frac{x}{5}$
   $x = 45 \times \frac{5}{x}$
   $x^2 = 225$
   x = <u>15 years old</u>

2. Since 5 is one of the numbers and there are no even numbers, then last digit must be <u>5</u>.

3. Suppose his average after n tests was 23, then his total after n tests was $23n$.
   Since his average after $n + 1$ tests was 22.25, then his new total was $22.25 \times (n + 1)$,
   or $22.25n + 22.25$.
   $23n$ + latest score = $22.25n + 22.25$
   Latest score = $22.25 - 0.75n$ or $22\frac{1}{4} - \frac{3}{4}n$
   $4 \times$ latest score = $89 - 3n$
   Latest score = $\frac{89-3n}{4}$ must be a multiple of 4, since n must be a whole number.
   For $\frac{89-3n}{4}$ to be between 18 and 30, $n = 3$
   Hence, his latest score was 22.25 – 2.25 = <u>20 marks</u>

4. For a number to be exactly divisible by 44, it must be divisible by both 4 and 11.
   A number is divisible by 4 if the number formed by its last two digits is a multiple of 4.
   There are 6 possibilities: 3764, 7364, 4736, 7436, 3476, and 4376
   A number is divisible by 11 if the difference of the sums of digits in the odd places and even places is 11 or 0. They are 7436 and 3476. So, only <u>2</u> numbers are divisible by 11.
   Ans: <u>2</u> numbers are divisible by 44.

5. Any square number can be written as the difference of two other squares numbers.
   The smallest 3-digit square number is <u>100</u>. (That is, $10^2 = 26^2 - 24^2$)

6. For any 20 consecutive numbers, there must be these composite numbers:
   10 numbers which are multiples of 2;
   3 numbers which are multiples of 3 but not multiples of two;
   1 number which is a multiple of 5 but not a multiple of 2 or 3.
   This means that there will be at least 14 numbers which are composite.
   Therefore, at most <u>6</u> numbers can be prime. (Example: 97, 101, 103, 107, 109, 113)

7. The tigers and lions have to be arranged alternatingly. Thus, L T L T L T L T L.
   The 5 lions can be arranged in $5 \times 4 \times 3 \times 2 \times 1 = 120$ ways.
   The 4 tigers can be arranged in $4 \times 3 \times 2 \times 1 = 24$ ways.
   Together, they can be arranged in 24×120 = <u>2880 ways</u>.

8. Since 7 units = 56 cm, then 3 units = 24 cm and 4 units = 32 cm
Since AC : BD is 3 : 4, then AO : BO is also 3: 4.
Then, AO : BO : AB must be 3 : 4 : 5 (Pythagoras theorem).
Since AO = 12 cm, then AB = 20 cm.
Thus, perimeter of rhombus = 20 × 4 = 80 cm

9. 1 litre = 10 cm × 10 cm × 10 cm = 1000 cm^3
1728 litres = 1728000 cm^3
The side of the cubic container = $\sqrt[3]{1728000}$ = 120 cm
120 ÷ 25 = 4 r 20
Only 4 × 4 × 4 = 64 cubic boxes can fit inside the cube container.

10. $\frac{x}{1} + \frac{1}{1} + \frac{x}{2} + \frac{2}{2} + \cdots + \frac{x}{100} + \frac{100}{100} = 100$
$\frac{x}{1} + 1 + \frac{x}{2} + 1 + \ ... + \frac{x}{100} + \ 1 = 100$
$x\left(1 + \frac{1}{2} + \cdots + \frac{1}{100}\right) + 100 = 100$
$x\left(1 + \frac{1}{2} + \cdots + \frac{1}{100}\right) = 0$
$\underline{x = 0}$

11. The second youngest child is 29 × 12 ÷ 3 = 116 months old. The sum of the ages of the three oldest children are 116 + (18 × 3) + 116 + (18 × 4) + 116 + (18 × 5) = 564 months. 564 ÷ 12 = 47 years old.

12. After the birds flew away, there was a total of 42 birds left. If n birds remained in the first tree then $2n$ remained in the second tree and $4n$ remained in the third tree. This gives a total of $7n$ birds remaining. Thus $7n = 42$ and so $n = 6$. In other words, after the birds flew away there were 6 left in the first tree, 12 in the second one, and 24 in the third one. Since 8 birds flew away from the second tree, there were originally 20 birds in the second tree.

13. Let m be the number of men and w the number of women in the group.
Then $200m + 160w = 2640$.
Thus $200m = 2640 - 160w$ or $5m = 66 - 4w$.
$66 - 4w$ must be a multiple of 5, hence w must be 4, 9 or 14; and m must be 10, 6 or 2 respectively. The only solution with an odd number of people is $w = 9$ and $m = 6$.
Therefore, there were 6 men in the group.

14. Let the middle number be x, which is between 27 and 49.
27 × 5 = 135 and 49 × 5 = 245
135 + 245 = 380 which includes x added twice.
$9x = 380 - x$
$10x = 380$
$x = 38$
The sum is 38 × 9 = 342

15. The digit in the Ones' place must be 5 and the digit in the Thousands' place must be 2 so that none of the digits in-between are 0. Hence, the digits in the Hundreds' and Tens' place are both 1.

16. If we multiply a number between 1 and 100 000 by 33 then we obtain a product between 33 and 3 300 000. So, if the product is to be *stuttering*, it must be either 111, 1 111, 11 111, 111 111 or 1 111 111. Among these numbers, only 111 and 111 111 are divisible by 3, and of these only 111 111 is also divisible by 11.
111 111 ÷ 33 = 3367
Thus, there is only 1 whole number between 1 and 100 000 which can be multiplied by 33 to give a *stuttering* product.

17. The worst-case scenario is to have 99 of each of the 20 colours. After that, if you have 1 more marble you would have 100 of one of the 20 colours. So, the least number is
$20 \times 99 + 1 = 1981$ marbles and that would weigh 1981×5g = 9905g

18. Since $\sqrt[3]{97336} = 46$, the huge cube has 46 small cubes along each side. If you remove all the small cubes which can be seen from the outside, there would remain $44 \times 44 \times 44$ = 85 184 cubes which could not have been seen from the outside.

19. $6x - 3y = 2x + 2y$
$4x = 5y$
$\frac{x}{y} = \frac{5}{4}$

20. $\frac{bc}{ab} = \frac{3}{2}$

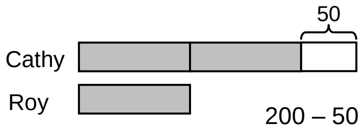
$\frac{de}{cd} = \frac{5}{4}$

$\frac{e}{a} = \frac{3}{2} \times \frac{5}{4} = \frac{15}{8}$

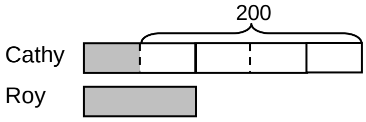
21. The total amount of sugar (in litres) in the mixture is $(2 \times 0.10) + (3 \times 0.15) = 0.65$.
The mixture itself has 5 litres of juice all together.
The percentage of sugar in the mixture is $\frac{0.65}{5} \times 100$ = 13%

22. Safe delivery of 500 pots → 500 × \$1.20
= \$600
Each pot damaged cost → \$1.20 + \$2 = \$3.20
600 – 568 = \$32
\$32 ÷ \$3.20 = 10 pots damaged.

23. If Cathy ate twice as many as Roy:



If Cathy ate half as many as Roy:



200 – 50 = 150
3 small units = 150
1 small unit = 50
Cathy : 50 × 4 + 50 = 250 candies
Roy : 50 × 2 = 100 candies

24. Let *AB* be the two-digit number, where *A* and *B* are digits from 1 to 9. The value of *AB* is 10*A* + *B*. When the digits are reversed, we obtain the number *BA*, with a value of 10*B* + *A*. To be *reflexive*, we must have (10*A* + B) − (10*B* + *A*) = 27.
This means that 10*A* + *B* – 10*B* − *A* = 27 and so 9*A* – 9*B* = 27 and *A* − *B* = 3.
The possibilities for *A* and *B*, respectively, are
4 and 1; 5 and 2; 6 and 3; 7 and 4 ; 8 and 5; and 9 and 6.
That is, the *reflexive* numbers are 41, 52, 63, 74, 85, and 96. There are 6 of them.

25. The first 3-digit odd number, exactly divisible by 3 is 105.
Subsequent numbers increase by 6.
The last number less than 456 is 453.
453 – 105 = 348
348 ÷ 6 = 58
58 + 1 = 59
There are 59 numbers.

26. In order for a number *n* to be fascinating, its digit in the Tens' place must be smaller than its digit in the Ones' place This is because the number with the digits reversed must be larger.
Also, *n* must be a multiple of 4. Otherwise, the number 75% larger will not be an integer.
Finally, *n* must be smaller than 58. Otherwise, the number 75% larger will have three digits.
The numbers that meet all three conditions are 12, 16, 24, 28, 36, 48, and 56. The ones
that are actually *fascinating* are 12, 24, 36, and 48. Thus, there are 4 such numbers.

27. The board is 8 by 8, with 4 black and 4 white squares in each row and each column.
Suppose you choose a black square first. There are 32 ways of doing this. For each way, you may choose any of the white squares which are not in the same row or column. Since
a total of 8 white squares will be in the same row or column as the black one, there will be 32 – 8 = 24 white ones to choose from. Hence, there are 32 × 24 = 768 ways of choosing
the two squares.

28. Area of MDNB is half Area of ABCD. Area of MPQD is half Area of MDNB.
Therefore, area of MPQD = (24 × 18) ÷ 4 = 108 cm^2

29. Total volume of cubes = 12 × 4 × 4 × 4 = 768 cm^3

$\frac{5}{6} - \frac{2}{3} = \frac{1}{6}$

Volume of cubes
= $\frac{1}{6}$ of height of container × 8 × 8 = 768 cm^3

$\frac{1}{6}$ of height of container = 768 ÷ 64
Height of container = 768 ÷ 64 × 6 = 72 cm

30. Distance Rachel travelled in 1.5 hrs = 84 × 1.5 = 126 km
Distance Rachel travelled after passing Eleanor = 126 + 39 = 165 km
Eleanor's speed = 165 ÷ 2.5 = 66 km/h
Distance between City F and City G = 4 × 66 = 264 km
The distance between the two cities was 264 km.